as benzoic acid, toluene, turpentine, &c. At a later date this severe method of treatment was replaced by the milder action of fused potash, with the result that a number of new aromatic acids and phenols were discovered. At the present time the separation of the various constituents of a resin is effected by the use of solvents and the numerous reagents which the resources of modern organic chemistry can offer. The results do not carry us very far. As the author says, "our march of conquest has only begun, and the present volume may suggest a successful plan of campaign." J. B. C.

The Lepidoptera of the British Islands: a Descriptive Account of the Families, Genera and Species Indigenous to Great Britain and Ireland, their Preparatory States, Habits and Localities. By Charles G. Barrett, F.E.S. Vol. vi. Parts 59-70. Heterocera (Noctuina—Geometrina). Pp. 388. Plates 233-280. (London: Lovell, Reeve and Co., Ltd., 1900.)

THE present instalment of Mr. Barrett's great work includes 110 species, from *Hoporina croceago*, Schiff., to *Halix wauaria*, L., and is written in the same exhaustive manner as previous volumes, giving all the information that a collector of British Lepidoptera (as such) is most likely to require. To Continental entomologists who wish to acquire an accurate knowledge of our limited insular fauna it would also prove very useful; though it is to be regretted that the bulk of the book, which may be expected to extend to nearly twenty volumes, and the unavoidable costliness of the larger edition issued with plates (which are not included in the cheap edition), must necessarily tend to restrict the sale. Those requiring it may therefore be recommended to obtain it volume by volume, or in monthly parts, as it appears, rather than to wait till the whole work is completed. We need not repeat our comments on earlier volumes, which will equally apply to the one before us; but the accounts given of the habits of the various moths discussed are always interesting, and sometimes curious; thus we learn that the rare *Cerastis erythrocephala*, Schiff., after its discovery in 1847, was met with occasionally till 1859, when it seems to have almost disappeared till 1872 and 1873, since when only one specimen, taken in 1894, has been found in England. The periodicity of the appearance of many species in these islands is curious, and has never been fully explained, for the causes which appear applicable to some cases will not explain others; and, moreover, uncertainty in the appearance of species seems to increase rather than to diminish. English names are not a conspicuous feature in this book, but Mr. Barrett notes that a recent writer has called *Xylina conformis*, Schiff., "The Conformist," and the next species, *X. lambda*, Fab., "The Nonconformist"! The resemblance of species of *Calocampa* and *Cucullia* to bits of stick is commented on; in fact, certain moths and larvæ thus fill the gap in our protected fauna caused by the absence of the stick insects proper, or Phasmidæ, which are not found nearer to our shores than the South of France. Several species noted in this volume seem to be now extinct in our islands; thus, *Chariclea delphinii*, L., does not seem to have been taken in England since about 1815. Their place has been taken by others; for example, the northern migration of *Plusia moneta*, Fab., reached England in 1890, and is probably still extending. Other moths of interest are those with cannibal larvæ, such as *Scopelosoma satellitia*, L., *Heliothis armigera*, Hübn., &c. There are many other interesting observations, which we have no room to quote, in the present volume, comprising, as it does, the conclusion of the Noctuæ, the Deltoidæ, and the first few species of the Geometræ. We may, however, note that the enigmatical *Sarrothripa revayana*, Schiff., is regarded by Mr. Barrett as a true *Noctua*, and is placed at the end of the *Noctuae Trifidae*.

W. F. K.

LETTER TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

The Plankton of the Bay of Biscay.

WITH the valuable assistance of Mr. L. A. Borradaile, of Selwyn College, Cambridge, I have just completed a series of observations on the plankton of the Bay of Biscay, extending over about three weeks, by means of opening and closing nets as well as ordinary tow-nets. Our observations point to the fact (unexpected at any rate by myself) that the smaller Mesoplankton practically ceases at a depth of about 1000 fathoms. This conclusion agrees with that reached by Prof. Chun on the basis of the *Valdivia* Expedition (Deutsche Tiefsee Expedition, 1898-99, p. 44), with which, however, we were unacquainted until we had arrived at it independently. Below this limit we almost always captured a few specimens, as to which it was doubtful whether they were alive when captured, or were merely corpses of a shallower fauna sinking to the bottom, but in a few cases we at present incline to assign them to a living Mesoplankton.

We have also taken about 90 hauls under varied conditions at varied depths between 100 fathoms and the surface, which will eventually, we hope, give a fairly accurate basis for the determination of the vertical movements of the Epiplankton.

Our thanks are due to the Lords Commissioners of the Admiralty for placing the ship at our disposal, and to Captain Field and the other officers of the *Research* for their ungrudging assistance. G. HERBERT FOWLER.

H.M.S. *Research*, Devonport, July 27.

THE TEACHING OF MATHEMATICS.

I THINK it very important to try to get a view of our system of teaching mathematics which is not too much tinted with the pleasant memories of one's youth. Like all the men who arrogate to themselves the right to preach on this subject, I was in my youth a keen geometrician, loving Euclid and abstract reasoning. But I have taught mathematics to the average boy at a public school, and this has enabled me to get a new view. I have seen faces bright outside my room become covered as with a thin film of dulness as they entered; I have known men, the best of their year in England in classics, lose in half an hour (as men did in the first day of slavery in old times) half their feeling of manhood; and I have known that, as an orthodox teacher of mathematics, I was really doing my best to destroy young souls. Happily, our English boys instinctively take to athletics as a remedy, and I know of nothing which gives greater proof of the inherent strength, in good instincts and common sense, of our race than this refusal to allow one's soul to be utterly destroyed. I have also mixed much with engineers, who really need some mathematics in their daily work, men who say that they once were taught mathematics, and I know that these men never use anything more advanced than arithmetic, and actually loathe a mathematical expression when it intrudes itself into a paper read before an engineering society. Of all branches of engineering, electrical engineering relies most upon exact calculation. Well, the average electrical engineer in good practice would rather work a week at many separate arithmetical examples than try for an hour to get out the simple algebraic expression, which includes all his week's results and much more. Yet he has passed perhaps certain rather advanced examinations in mathematics. Furthermore, those engineers who can most readily apply mathematics to engineering problems, almost invariably descend to the position of teachers and professors in schools and colleges, and they seem to lose touch completely with the actual life of their profession.

I have studied these phenomena very carefully, and I affirm that they are directly traceable to the absurd thing called mathematical teaching in schools and colleges.

The framers of educational methods took in their youth to abstract reasoning as a duck takes to water, and of course they assume that a boy who cannot in one year understand a little Euclid must be stupid. In truth, it is a very exceptional mind, and not, perhaps, a very healthy mind, which can learn things or train itself through abstract reasoning; nor, indeed, is much ever learnt in this way. Do we philosophise about swimming before we know how to swim; or about walking or jumping, or cycling or riding a horse, or planing wood, or chipping or filing metals, or about playing billiards or cricket? Is it through philosophy that we learn a game of cards, or to read or to write? No; we first learn by actual trial; we practice as our mind lets us; we philosophise afterwards—perhaps long afterwards. Then if we are too clever or stupid, we insist on teaching a pupil from the point of view which we have at the end of our studies, and we refuse to look at things from the pupil's point of view.

What a natural but ghastly statement the boy made who said: "Yes, Euclid and Xenophon, the beasts, wrote books for the third and fourth forms"! It is even a ghastlier notion that the jokesomeness of a philosopher, the unessential fringe of a subject, often becomes the soul-destroying, weary, worrying study of a schoolboy.

In a short article I shall not attempt to put forward my views as to how mathematics ought to be taught; I have published some of them in a summary of lectures on "Practical Mathematics," published by the Science and Art Department, and in my "Calculus for Engineers."

We let a Board School boy jump over all the ancient philosophy of arithmetic with its twenty-seven independent Greek characters (for our ten figures), the study of which required a lifetime, so that only old men could do multiplication, and they not only needed many hours to do one easy bit of multiplication, but declared that if the art were not practised every day it could not be remembered. Why not also let a boy jump over all the Euclidian philosophy of geometry, and assume even the forty-seventh proposition of the first book of Euclid to be true? Why not let him replace the second and fifth books of Euclid by a page of simple algebra, and give him much of the sixth book as axiomatic? If you must insist on abstract reasoning, you had better remember that nothing is really axiomatic; but any well-established truths may be looked upon as fundamental or axiomatic, and a system of abstract reasoning may be founded upon them. At present, a man at Cambridge finishes just where the really interesting and useful part of mathematics begins. There would not indeed be much difficulty in framing a course in which he would begin by studies where the studies of good mathematicians now end. This has been tried and proved successful. The present rules of the game are really a little too absurd. A difficult vector subject like geometry must be studied before algebra. Simple exercises on squared paper, well within the capacity of even illiterate persons, must not be approached until one has wasted years on higher algebra and trigonometry and geometrical conics, because they belong to the subject of co-ordinate geometry. It is assumed that it is not until after co-ordinate geometry is thoroughly studied that a man can take in the idea which underlies the calculus, an idea which is possessed by every young boy with absolute accuracy, and by every healthy mind.

Some friends of mine assert that no boy or man ought to be allowed to use logarithms until he knows how to calculate logarithms. They say this, knowing that the calculation is a branch of what is called higher mathematics, and that the average schoolboy, after six years at mathematics, finds it hopeless to even begin the study of the exponential theorem. It is a hard saying! It is exactly like saying that a boy must not wear a watch or a pair of trousers until he is able to make a watch or a pair of trousers. I am an advocate for the use by all students of all appliances which may be useful to them, whether made by tailors, or watchmakers, or instrument makers, or builders, or pure mathematicians. We need not believe a craftsman when he tells us that we cannot utilise his results without practising his trade. Nevertheless, it is good to be able to do some things for one's self, such as sewing on buttons, or using the lathe or a blowpipe, or the development of a little mathematics. If readers will refer to the above-mentioned *summary* they will see that I consider a good system of mathematics teaching of fundamental importance in the education of all men.

I must not dwell any longer on the imperfections of the existing system, but I hope that even readers who do not quite agree with me that much of the sixth book of Euclid ought to be regarded as axiomatic, will agree that what we usually call arithmetic is useless. For races not troubled with our abominable system of weights and measures, the whole of arithmetic consists merely of multiplication and division. To them a decimal is no more difficult to understand than an ordinary number. It is supposed that an English boy understands at once the meaning of 4,590,000 or 4590 or 459, but that such a number as 45·9 or ·459 or ·00459 is beyond his comprehension. I say that this is a difficulty artificially maintained by our stupid methods of teaching. Like the rest of our stupid methods, it is due to our unscientific ways of thinking. Because the embryo passes through all the stages of development of its ancestors, a boy of the nineteenth century must be taught according to all the systems ever in use and in the same order of time. The decimal system of stating numbers is 700 years old in Europe, but it was not till 280 years ago that Napier invented the use of decimals and the decimal point. Think of compelling all emigrants to pass to America through Cuba, because Cuba was discovered first. Think of making boys learn Latin and Greek before they can write English, because Latin and Greek were the only languages in which there was a literature known to Englishmen 450 years ago.

Again, the ingenious teachers of last century incorporated every kind of arithmetical example in a book and called each kind by a slang name—practice, interest, discount, tare and tret, alligation, position, &c., and we must teach exactly as they did. I do not mind retaining the buttons at the back of my coat; many useless ancient ceremonies may still be practised, and I shall not object. I can even admire them, but the unscientific waste of the valuable youth of millions of our people, now that we are face to face with nations who are determined to destroy England through commerce and war, is so abhorent to me that I cannot think of it with patience. It is not merely in arithmetic and geometry, but in the higher parts of mathematics that this waste goes on. Newton employed geometrical conics in his astronomical studies, and mechanics was developed; and therefore it is that every young engineer must study mechanics through astronomy, and he dare not think of the differential calculus till he has finished geometrical conics. The young applier of physics, the engineer, needs a teaching of mathematics which will make his mathematical knowledge part of his mental machinery, which he shall use as readily and certainly as a bird uses its wings; and we teach him in such a way that he hates the sight of a mathematical symbol all his life after.

It is just as in classics. Ask the average man if he ever reads anything now in Latin or Greek; ask him about anything to which he devoted ten years of his study at school, and he will answer that the only men he knows of who read the classics are a few famous scholars and the cads who read with delight cribs of the *Odyssey* and the *Iliad* just as if they were novels, because they never had the advantage of a classical education. But, of course, his mind was trained, he can always say that.

The authorities of the Science and Art Department recognise that apprentices and others attending evening classes may possibly benefit by a course of study very different from what is necessary if students are being prepared for university and other examinations. Hence, in addition to their very complete orthodox courses of instruction, they recognise the new method of study, the most elementary part of which is beginning to get crystallised in the following syllabus. There is also an "Advanced" syllabus, which is too long to be published here. I would advise interested persons to write to the Department for copies, and also for the report on the result of last year's examination, as well as for copies of the examination papers and of the above-mentioned summary.

I venture to hope for criticism of this syllabus—first, from men like my Cambridge friends, who are quite sympathetic, but who think the method one fit for evening classes only; second, from men who think with me that the method is one which may be adopted in every school in the country, and adopted even with the one or two boys in a thousand who are likely to become able mathematicians; third, from other men. Whatever be the point of view of any critic, he must surely feel that exhaustive criticism is important, for there are many large technical schools in England in which the method has already been adopted, the orthodox system being quite given up. I have been informed that the method is spreading rapidly in Germany also. I can already see from the exceedingly interesting examination results that crystallisation is proceeding rapidly, and if criticism is to be of value, now is the time for it. I hope also that the seemingly bumptious manner in which I criticise orthodox methods of teaching will not induce contemptuous indifference in men of thought. I hold a brief in the interests of average boys and men; my strong language and possible excess of zeal are due to the fact that nearly all the clever men have briefs on the other side.

JOHN PERRY.

PRACTICAL MATHEMATICS.

ELEMENTARY STAGE.

Arithmetic.—The use of decimals; the fallacy of retaining more figures than are justifiable in calculations involving numbers which represent observed or measured quantities. Contracted and approximate methods of multiplying and dividing numbers whereby all unnecessary figures may be omitted. Using rough checks in arithmetical work, especially with regard to the position of the decimal point.

The use of $5{\cdot}204 \times 10^5$ for 520400 and of $5{\cdot}204 \times 10^{-3}$ for ·005204. The meaning of a common logarithm; the use of logarithms in making calculations involving multiplication, division, involution and evolution. Calculation of numerical values from all sorts of formulæ.

The principle underlying the construction and method of using a common slide rule; the use of a slide rule in making calculations. Conversion of common logarithms into Napierian logarithms. The calculation of square roots by the ordinary arithmetical method. Using algebraic formulæ in working questions on ratio and variation.

Algebra.—To understand any formula so as to be able to use it if numerical values are given for the various quantities. Rules of Indices.

Being told in words how to deal arithmetically with a quantity, to be able to state the matter algebraically. Problems leading to easy equations in one or two unknowns. Easy transformations and simplifications of formulæ. The determination of the numerical values of constants in equations of known form, when particular values of the variables are given. The meaning of the expression "A varies as B."

Factors of such expressions as x^2-a^2, $x^2+11x+30$, $x^2-5x-66$.

Mensuration.—The rule for the length of the circumference of a circle. The rules for the areas of a triangle, rectangle, parallelogram, circle; areas of the surfaces of a right circular cylinder, right circular cone, sphere, circular anchor ring. The determination of the area of an irregular plane figure (1) by using a planimeter; (2) by using Simpson's or other well-known rules for the case where a number of equidistant ordinates or widths are given; (3) by the use of squared paper whether the given ordinates or widths are equidistant or not, the "mid-ordinate rule" being used. Determination of volumes of a prism or cylinder, cone, sphere, circular anchor ring.

The determination of the volume of an irregular solid by each of the *three* methods for an irregular area, the process being first to obtain an irregular plane figure in which the varying *ordinates* or widths represent the varying *cross sections* of the solid.

Some practical methods of finding areas and volumes. Determination of weights from volumes when densities are given.

Stating a mensuration rule as an algebraic formula. In such a formula any one of the quantities may be the unknown one, the others being known.

Use of Squared Paper.—The use of squared paper by merchants and others to show at a glance the rise and fall of prices, of temperature, of the tide, &c. The use of squared paper should be illustrated by the working of many kinds of exercises, but it should be pointed out that there is a general idea underlying them all. The following may be mentioned:—

Plotting of statistics of any kind whatsoever, of general or special interest. What such curves teach. Rates of increase.

Interpolation, or the finding of probable intermediate values. Probable errors of observation. Forming complete price lists by shopkeepers. The calculation of a table of logarithms. Finding an average value. Areas and volumes, as explained above. The method of fixing the position of a point in a plane; the x and y and also the r and θ, co-ordinates of a point. Plotting of functions, such as $y=ax^n$, $y=ae^{bx}$, where a, b, n, may have all sorts of values. The straight line. Determination of maximum aud minimum values. The solution of equations. Very clear notions of what we mean by the roots of equations may be obtained by the use of squared paper. Rates of increase. Speed of a body. Determination of laws which exist between observed quantities, especially of linear laws. Corrections for errors of observation when the plotted quantities are the results of experiment.

In all the work on squared paper a student should be made to understand that an exercise is not completed until the scales and the names of the plotted quantities are clearly indicated on the paper. Also that those scales should be avoided which are obviously inconvenient. Finally, the scales should be chosen so that the plotted figure shall occupy the greater part of the sheet of paper; at any rate, the figure should not be crowded in one corner of the paper.

Geometry.—Dividing lines into parts in given proportions, and other illustrations of the 6th Book of Euclid. Measurement of angles in degrees and radians. The definitions of the sine, cosine and tangent of an angle; determination of their values by drawing and measurement; setting out of angles by means of a protractor when they are given in degrees or radians, also when the value of the sine, cosine or tangent is given. Use of tables of sines, cosines and tangents. The solution of a right angled triangle by calculation and by drawing to scale. The construction of a triangle from given data; determination of the area of a triangle. The more important propositions of Euclid may be illustrated by actual drawing; if the proposition is about angles, these may be measured by means of a protractor; or if it refers to the equality of lines, areas or ratios, lengths may be measured by a scale and the necessary calculations made arithmetically. This combination of drawing and arithmetical calculation may be freely used to *illustrate* the truth of a proposition.

The method of representing the position of a point in space by its distances from three co-ordinate planes. How the angles are measured between (1) a line and plane; (2) two planes. The angle between two lines has a meaning whether they do or do not meet. What is meant by the projection of a line or a plane figure on a plane. Plan and elevation of a line which is inclined at given angles to the co-ordinate planes. The meaning of the terms "trace of a line," "trace of a plane."

The difference between a *scalar* quantity and a *vector* quantity. Addition and subtraction of vectors.

Slope of a line; slope of a curve at any point in it. Rate of increase of one quantity y relatively to the increase of another quantity x; the symbol for this rate of increase, namely, $\frac{dy}{dx}$; how to determine $\frac{dy}{dx}$ when the law connecting x and y is of the form $y=ax^n$. Easy exercises on this rule.

In setting out the above syllabus the items have been arranged under the various branches of the subject.

It will be obvious that it is not intended that these should be studied in the order in which they appear; the teacher will arrange a mixed course such as seems to him best for the class of students with whom he has to deal.

ANALYTICAL PORTRAITURE.

IT seems well to put on record the principal results of experiments that I have recently made to *isolate the particulars* in which one portrait differs from another. They had a measure of success, but not enough to deserve illustration or lengthy description. The objects I had hoped to attain are important; namely, to define photographically the direction and degrees in which any individual differs from the race to which he belongs, the race being represented by a composite picture of many individuals belonging to it. Or, again, to define the particulars in which any variety of a plant or animal differs from its parent species. Or to define family features; or to isolate expressions, recollecting that these consist both of subtractions from, and additions to, the features as seen in repose.

My starting point was that the exact superimposition of a rather faint positive upon its rather faint negative produces an approximately uniform grey, when they are viewed as a single transparency. Thus, I photographed a rotating disc that had been faced with white paper and divided into concentric rings. The innermost disc was left white, the outermost ring was painted black, and the intermediate rings contained successively increasing proportions of black to white. The photographic negative showed rings of graded tints, and from this I took a positive by contact. Subsequently applying the positive to the negative, film to film, and viewing them as a transparency, a nearly uniform grey surface was produced. It was necessary to superimpose them with exactness; otherwise the edges of the rings were conspicuously dark in one part, and light in the opposite part. Another test experiment was to paste together thicknesses of tracing paper—two-fold, three-fold, &c., up to twelve-fold—to cut distinctively shaped snippets of these and to variously distribute them over the surface of a glass plate, which was then photographed, and a positive taken as well. On treating the positive and negative as above, all the tints between those of the three-fold and the nine-fold inclusive produced a uniform grey.

Let A and B be any two pictures whose respective negatives and positives will be called *neg. a*, *pos. a*, *neg. b*, *pos. b*. My object was to produce photographically a third pictue X which should express the difference between A and B; that is, should be equal to A—B, or else a fourth picture Y which should represent B—A.

It will, however, be simpler to treat the problem at first as an optical one, based on the following equations:—

$$\text{(I.) } pos.\ a + neg.\ a = grey;\ \text{(II.) } pos.\ a + x = pos.\ b$$

(if treated as a photographic problem, (II.) would be replaced by $pos.\ a + x = neg.\ b$). From these we obtain

$$\text{(III.) } pos.\ a + \{pos.\ b + neg.\ a\} = pos.\ b + grey$$

and

$$\text{(IV.) } pos.\ b + \{pos.\ a + neg.\ b\} = pos.\ a + grey.$$

Calling the terms within brackets by the name of "transformers," the transformer of b into a is the negative of the transformer of a into b. The two terms within brackets may be "composited" together on equal terms, then the result may be composited with the first term, allowing two-thirds of the total time of exposure to the transformer, and one-third to the first term. Or, what comes to the same thing in the end, all three terms may be composited in equal shares, allowing one-third of the total time of exposure to each. The transformers in (III.) and (IV.) being respectively $x + grey$ and $y + grey$, are nearly equivalent for the purposes of the inquiry to x and y, because the addition of a uniform shade of grey has little or no effect on pictorial resemblance. A portrait does not cease to resemble the original when it has become somewhat browned by exposure to a London atmosphere, or when it is viewed in shade, or under a tinted glass. Its distinctiveness depends on the *differences* (not the ratios) being preserved between the tints of all adjacent elements of its surface. Of course the grey must not be too dark; otherwise the deeper tints of the portrait would appear indistinguishably black.

This method of transformation succeeds fairly well. I changed an F on a white ground into a good G on a grey ground, and I changed with passable success one portrait A on white ground into another portrait B on grey ground, but the transformer itself gave little of that information to the eye which I had expected. It *must* have nearly isolated, but it failed to exhibit in an intelligible form the differences between A and B. Then I photographed two faces, each in two expressions, the one glum and the other smiling broadly. I could turn the glum face into the smiling one, or *vice versa*, by means of the suitable transformer; but the transformers themselves were ghastly to look at, and did not at all give the impression of a detached smile or of a detached glumness.

Part of the ghastliness was due to the different densities of the superimposed positives and negatives, which did not neatly obliterate one another in the unchanged portions of the face, and part was due to their not being superimposed in the best possible way. There can be no doubt of the best fit when engaged in making the transformer of an I into an L; but the eye must determine the best fit and proportions of the two components of the transformer of one portrait into another. I cannot yet make up my mind whether or no the process admits of substantial improvement, but feel sure that the only satisfactory experiments now would be those made by two converging lanterns on a screen, one at least of which admits of easy and delicate adjustment in direction and in the intensity of its illumination. The most suitable portraits for the attempt are apparently such as are popularly, and sometimes reproachfully, termed "artistic," that is to say, with blurred outlines and medium tints; certainly not those which in photographic language are called "plucky." I have no means in my house for experiments of this kind, but perhaps a trial might be made in some laboratory where they exist. The point is to ascertain whether the images of *neg. a* and *pos. b* can be so combined on the screen as to give an intelligible and useful idea of the differences between A and B.

FRANCIS GALTON.

A RECOLLECTION OF KING UMBERTO.

HOW enthusiastically the late King of Italy could devote himself to the welfare of science and art, those of us who were at Como last September had an opportunity of seeing. One very hot day he arrived with the Queen and the Duke of Naples by train from their palace at Monza, near Milan. First they made an official inspection of the galleries and machinery in the Silk and Electricity Exhibition, then they visited the Exhibition of Sacred Art, and, after lunch, they opened